some Ch 18 & 19 - orbit & spin ang mom

$L \quad S \quad (\text{total} = J)$

Q.M. $\rightarrow$ 3 dim results

functions $Y_{l, m_L}(\theta, \phi) \rightarrow$ spherical harmonics

l = orb ang mom quantum #

m_L = z comp of orb ang mom quantum #

$$\text{Ang mom} = \left[\sqrt{l(l+1)}\right] \hbar \qquad l = 0, 1, 2, \ldots$$

L = magnitude of

Energy depends on l, m_l if mag fields (internal or external)

$$L_z = m_L \hbar$$

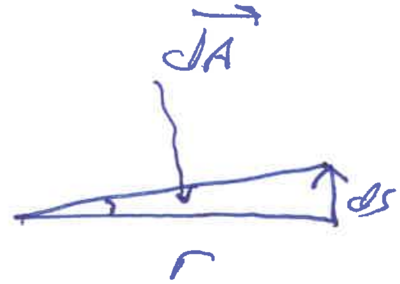
z comp

$m_l = -l$ to $+l$

$$\cos\theta = \frac{L_z}{L}$$

only allowed θ_s

Classical refresher



$$\vec{L} = \vec{r} \times \vec{p}$$

$$= m\vec{r} \times \frac{d\vec{s}}{dt}$$

$$= m \frac{(2 d\vec{A})}{dt}$$ ---- recall cross product is area of ||-ogram

If $\vec{L}$ is const (no torques)

$$= 2m \vec{A}/T$$

Now consider the mag dip mom μ_ℓ for e^- in this loop

$$\vec{\mu}_\ell = i\vec{A} = -\frac{e}{T} \cdot \vec{A}$$

relate

$$\mu_L = \left(-\frac{e}{2m}\right) L$$

Classical relation between μ, L.
In Q.M. we might expect different
But lets try classical relation

$$|\mu_l| = \frac{e}{2m}\sqrt{l(l+1)}\,\hbar$$

$$= \frac{e\hbar}{2m}\sqrt{l(l+1)}$$

$l = 0$ to $n-1$

$$\mu_{l_z} = -\frac{e\hbar}{2m}\, m_l$$

$m_l = -l$ to l

$$\frac{e\hbar}{2m} = \text{Bohr magneton}$$

$$= \mu_B$$

$$= 9.274 \times 10^{-24}\ J/T$$

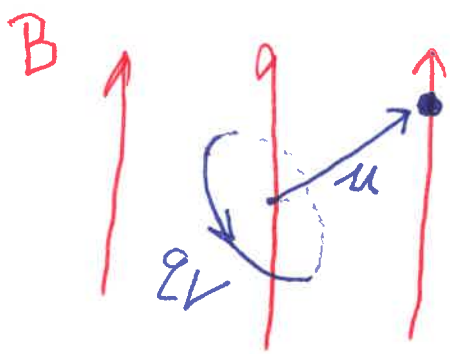
$$= 5.788 \times 10^{-5}\ eV/T$$

Classical refresh

$$\vec{\tau} = \overrightarrow{\text{dipole mom}} \otimes \overrightarrow{\text{field strength}}$$

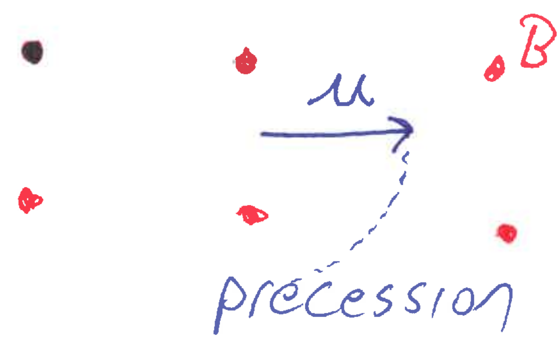
for grav
Elec
Mag
⋮

$$= \vec{\mu} \otimes \vec{B} \qquad \text{RHR}$$



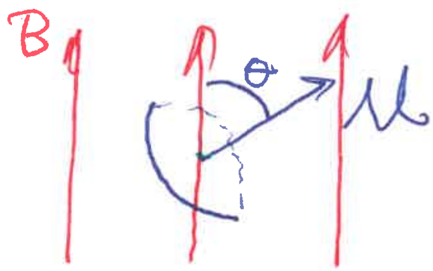
τ is out of page RHR

eyeball view



If there were no ang mom, then the dipole would swing into alignment with $\vec{B}$. A non-spinning top falls. A top with "L" precesses around field. For a top we can change "L" and dip mom indep. Not so with orbiting electron or spin! Also Ang mom conserved! precession!

Energy $\rightarrow$ $U = -\vec{\mu} \cdot \vec{B}$

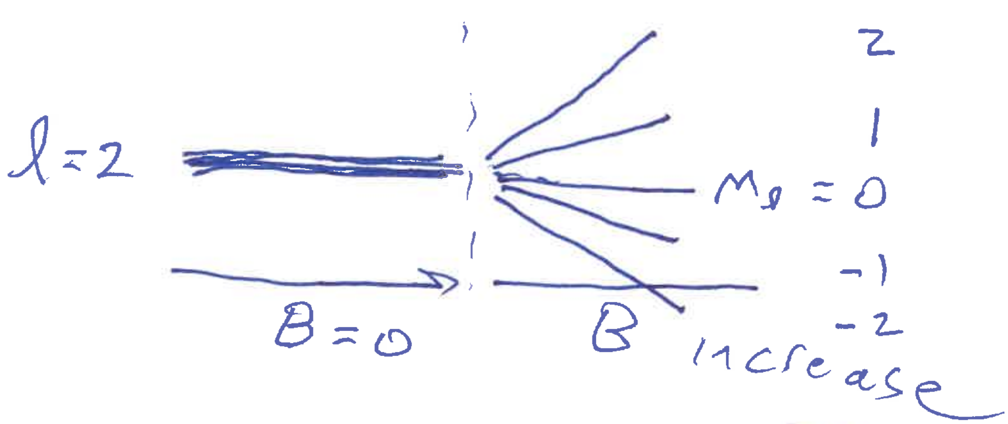


we have defined zero at $\perp$ or $90°$ & lowest energy is aligned.

We consider B to define the z axis so need to look at only $\mu_{m_l} \rightarrow$ z comp —

$$U = \mu_B \, m_l \, B$$

so each m_l level in B splits to different energy



Zeeman effect

Transitions $\Delta l = \pm 1$

$\Delta m_l = 0, \pm 1$ } selection rules!

Light carries....

SPIN

Neutrons, protons, & electrons (quarks too) have intrinsic angular mom $\vec{S}$ called spin.

$$\vec{S} = \sqrt{s(s+1)}\ \hbar$$

where $s = \frac{1}{2}$ for electrons (proton, neutron)

Total spins can add up for many particle systems.

$$\vec{S}_z = m_s \hbar = \pm \frac{1}{2} \hbar$$

The mag dip moment relates to spin ang momentum.

QED

$$\mu_s = -2.\ldots \frac{e}{2m} \vec{S}$$

$$= -2.\ \mu_B \sqrt{s(s+1)}$$

$$\mu_{s_z} = \mp 1.\ \mu_B$$

Again energy coupling & charges

The angular. mom – orbital $\vec{L}$ and spin $\vec{S}$ both add together to form total $\vec{J} = \vec{S} + \vec{L}$

There is an energy interaction between magnets μ_s & μ_L.

Quantum #s

$\cancel{L, S}$ $\quad |l, s, \underbrace{m_l, m_s}\rangle \to |l, s, \underbrace{j, m_j}\rangle$

For weak $\vec{B}_{ext}$, energies dep on

One may think of zero external field, so μ_s & μ_L couple!

As B_{ext} turns on, weak, μ_s & μ_L still couple to form $J \ldots \mu_J$.

As B_{ext} becomes strong μ_s & μ_L become decoupled — rather than couple together — each is coupled to B_{ext}